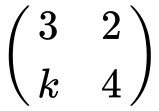
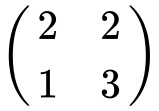
# 1.14 Introduction to matrices\_P\_1

**1a.** *[3 marks]*

Let ***A*** =  and ***B*** = . Find, in terms of ,

2***A***− ***B***.

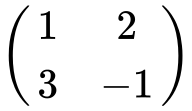
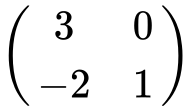


**1b.** *[3 marks]*

det (2***A***− ***B***).



**2a.** *[2 marks]*

Let ***A*** =  and ***B*** = .

Find ***A*** + ***B***.



**2b.** *[2 marks]*

Find −3***A***.

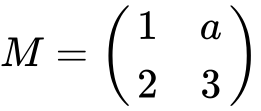
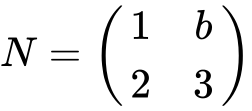


**2c.** *[3 marks]*

Find ***AB***.



**3a.** *[4 marks]*

Find a relationship between  and  if the matrices  and  commute under matrix multiplication.



**3b.** *[2 marks]*

Find the value of  if the determinant of matrix  is −1.

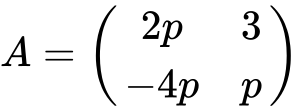


**3c.** *[1 mark]*

Write down  for this value of .

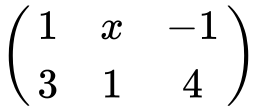
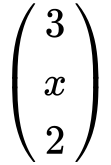


**4.** *[4 marks]*

If  and det , find the possible values of .



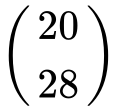
**5a.** *[3 marks]*

Let ***A*** =  and ***B*** = .

Find ***AB***.

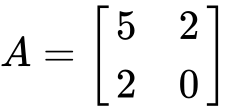
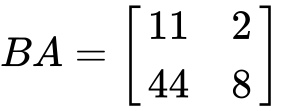


**5b.** *[3 marks]*

The matrix ***C*** =  and 2***AB*** = ***C***. Find the value of .

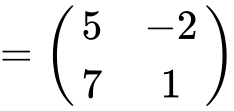


**6.** *[4 marks]*

 and  are 2 × 2 matrices, where  and . Find 



**7a.** *[2 marks]*

Consider the matrix ***A*** .

Write down the inverse, ***A***.



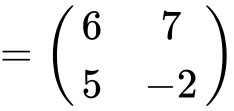
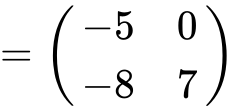
**7b.** *[2 marks]*

***B***, ***C*** and ***X*** are also 2 × 2 matrices.

Given that ***XA*** + ***B*** = ***C***, express ***X*** in terms of ***A***, ***B*** and ***C****.*

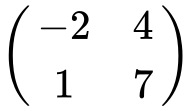
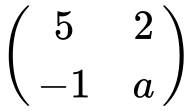


**7c.** *[2 marks]*

Given that ***B*** , and ***C*** , find ***X***.



**8a.** *[3 marks]*

Let *C* =  and ***D*** = .

The 2 × 2 matrix ***Q*** is such that 3***Q*** = 2***C*** – ***D***

Find ***Q***.



**8b.** *[4 marks]*

Find ***CD***.

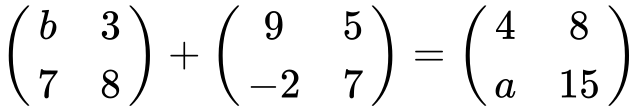


**8c.** *[2 marks]*

Find ***D***.



**9a.** *[1 mark]*

Let .

Write down the value of .

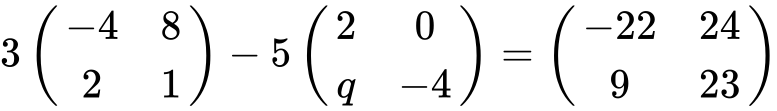


**9b.** *[2 marks]*

Find the value of .



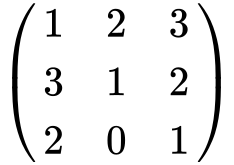
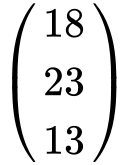
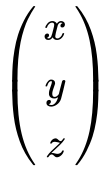
**9c.** *[3 marks]*

Let .

Find the value of .



**10a.** *[2 marks]*

Let ***A*** = , ***B*** = , and ***X*** = .

Write down the inverse matrix ***A***.



**10b.** *[1 mark]*

Consider the equation ***AX*** = ***B***.

Express ***X*** in terms of ***A*** and ***B***.

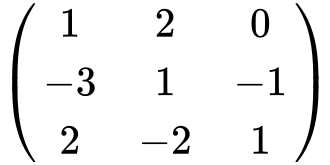
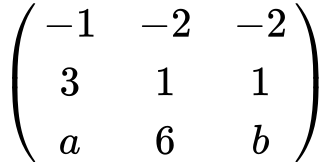


**10c.** *[3 marks]*

**Hence**, solve for ***X***.



**11a.** *[1 mark]*

The matrix ***A*** =  has inverse ***A*** = .

Write down the value of .



**11b.** *[1 mark]*

Write down the value of .



**11c.** *[1 mark]*

Consider the simultaneous equations







Write these equations as a matrix equation.



**11d.** *[3 marks]*

Solve the matrix equation.



**12a.** *[7 marks]*

The function *f* : ***M*** → ***M*** where ***M*** is the set of 2 × 2 matrices, is given by *f*(***X***) = ***AX*** where ***A*** is a 2 × 2 matrix.

Given that ***A*** is non-singular, prove that *f* is a bijection.



**12b.** *[4 marks]*

It is now given that ***A*** is singular.

By considering appropriate determinants, prove that *f* is not a bijection.



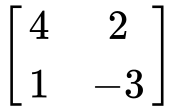
**13a.** *[3 marks]*

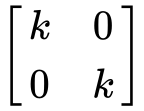
Let ***A*** = 2***A*** + ***I*** where ***A*** is a 2 × 2 matrix.

Show that ***A*** = 12***A*** + 5***I***.



**13b.** *[3 marks]*

Let ***B*** = .

Given that ***B*** – ***B*** – 4***I*** = , find the value of .



**14a.** *[8 marks]*

Consider the simultaneous linear equations


where  and  are constants.

Using row reduction, find the solutions in terms of  and  when  ≠ 3 .



**14b.** *[1 mark]*

Explain why the equations have no unique solution when  = 3.



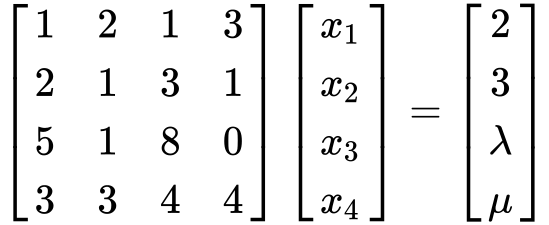
**14c.** *[4 marks]*

Find all the solutions to the equations when  = 3,  = 10 in the form ***r*** = ***s*** + ***t***.



**15a.** *[5 marks]*

Consider the system of equations



Determine the value of  and the value of  for which the equations are consistent.

**15b.** *[3 marks]*

For these values of  and , solve the equations.

**15c.** *[2 marks]*

State the rank of the matrix of coefficients, justifying your answer.

**16.** *[4 marks]*

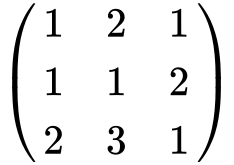
A matrix ***M*** is called idempotent if ***M*** ***M***.

(i)     Explain why ***M*** is a square matrix.

(ii)     Find the set of possible values of det(***M***).



**17.** *[12 marks]*

The matrix ***A*** is given by ***A*** = .

(a)     Given that ***A*** can be expressed in the form ***A******A******A*** ***I***, determine the values of the constants , , .

(b)     (i)     Hence express ***A*** in the form ***A******A******A*** ***I*** where .

(ii)     Use this result to determine ***A***.

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